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Fluctuation of windstorm premiums for Excess of Loss reinsurance in Japan

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This article is a summary of my examination paper for my Bachelor of Science (Mathematical statistics) describing a project that I have done for Skandia International during my last year at Stockholm University.

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1. Preface

The purpose of the project was to build a mathematical model that describes the premium fluctuation in windstorm reinsurance (Japanese Windstorm Catastrophe Excess of Loss). To do so, I have defined a Japanese Market Rate Index and calculated the expected loss and variance for a layer, when the losses are Pareto distributed and truncated at a maximum.

My conclusion is that it is impossible to statistically build a mathematical model and verify all the parameters for the premium fluctuations with too little information at hand. In this project I have only been able to use a small data material but since the reinsurance market is global it should be possible, with additional data material, to build a model of the global catastrophe premiums.

The article is divided in a number of sections starting with Background and after that Information, Decision of parameter to analyse, Distribution adjustment, Calculation of pure premium, Mathematical model of rate variations and finally Analysis results.

I have assumed that the reader has a good knowledge of reinsurance, especially Excess of Loss reinsurance.

2. Background and description of the project

Skandia International has since 1992 written Windstorm Excess of Loss (per event) treaties from different Japanese ceding companies. In 1991 there was a large windstorm called Mireille (T-19) that resulted in large losses for insurers and reinsurers. The losses were in the same magnitude as all storms together since the beginning of the 80ths. There has not been any large storm in Japan since then, but there has been other large storms in other places, e.g. Andrew, which hit the USA in 1992.

Skandia International's personnel in Japan have stated that the Japanese market is payback oriented and wants a long-term business connection. They also say that at least in the past, any big loss including Mireille have fully been paid back in a very short period of time. Despite the fact this has proven effective in the past, it can not be taken for granted that the precedent will continue, especially when the regulations of the Japanese insurance market has been decreasing since 1991.

The premiums for all Excess of Loss layers were high the first two years after Mireille, but since then they have dropped to approximately half of the 1992/1993 level, and the big question for Skandia International is how long they should accept these premium reductions.

Skandia International put up the following purposes for the project:

- Set up a mathematical model for how the reinsurance premium for Windstorm Excess of Loss treaties fluctuates over time; i.e. establish relation between the market premium and the last major windstorm.
- Set up a simulation routine, to calculate the long-term result for Japanese reinsurance contracts.

3. Information

Before 1992 windstorm was reinsured together with earthquake under catastrophe reinsurance so the analysable data is only available for the years 1992 to 1997. The information consists of 29 contracts from 12 companies.

In the analysis I have used a loss distribution provided by Skandia International and for gross losses, i.e. before deduction of any proportional reinsurance, facultative reinsurance, or any per risk Excess of Loss reinsurance. This distribution has a maximum loss amount 9 times greater than the loss Mireille (T-19 1991) caused the ceding companies, so the distribution is truncated.

3.1 Choice of parameters to analyse

The big question is how the premiums fluctuate over years. Since the contracts change from one year to another, we will need a parameter, which take the risk and the premium into consideration.

When the pure premium is calculated I have used Mireille and subject premium (protected premium base) as references. The Adjustment Rate takes subject premium changes into consideration and eliminate the effect of inflation. That is why I have used a ratio between the actual layer price (Adjustment rate) and the technical pure premium (Pure Rate) to look at the fluctuation over time. This ratio, denoted $Q_i(t)$, also makes contracts and layers comparable.

Definition: Market Rate Index (*MRI*)

The charge that the Pure Rate shall be multiplied with to get the Adjustment Rate in the Japanese market is denoted the Market Rate Index, i.e. the ratio between the Adjustment Rate and Pure Rate.

$$MRI(t) = \frac{\text{The Rate year t}}{\text{The Pure Rate year t}}$$

Every ratio $Q_i(t)$ is a sample of the random parameter Japanese Market Rate Index and a good estimation of the $MRI(t)$ is the average ratio of all the contracts for each year, i.e.

$$M\hat{R}I(t) = \frac{1}{n}\sum_{i=1}^{n} Q_i(t) =$$

$$= \frac{1}{n}\sum_{i=1}^{n} \frac{\text{The Rate year t for contract i}}{\text{The Pure Rate year t for contract i}}$$

4. Distribution adjustment

The information of the loss amount that I got from Skandia International is the empirical distribution function (*EF*). To this *EF*, I need to adjust a distribution function with a "heavy tail", e.g. the Pareto distribution or the Log-normal distribution, truncated at 28.

In the table below we can see for example that a loss of size 2.333 or smaller will occur with the probability of 0.9 i.e. once every 10th year we will have a loss 2.333 or worse.

Skandia International's distribution of losses

Loss size	*EF (x)*	*Year*
1.000	0.667	3
1.333	0.750	4
1.667	0.800	5
2.000	0.875	8
2.333	0.900	10
2.667	0.917	12
3.333	0.950	20
5.000	0.960	25
7.333	0.975	40
10.000	0.980	50
15.000	0.990	100
20.000	0.995	200
23.333	0.998	500
26.667	0.999	1000

4.1 Adjustment measure

It is not so common to adjust distributions to an empirical distribution function without any observations so some sort of measure of adjustment is needed. There are a couple of different measures that can be used and I have chosen this:

Minimise the differences between the empirical distribution and the parametric distribution in both vertical and horizontal way at the same time. Finish the adjustment with a check that storms of Mireille's magnitude or worse occurs with the same frequency and that the pure (technical) premium is approximately the same.

To compare the difference between the pure premiums I used the formula that is calculated in the next section and the following estimated premium (for layer 3 excess of 1).

$$\text{Premium for layer} = P(1 \le X \le 1{,}333) \cdot \left(\frac{(1+1{,}333)}{2} - 1 \right) +$$
$$+ P(1{,}333 \le X \le 1{,}667) \cdot \left(\frac{(1{,}333+1{,}667)}{2} - 1 \right) + \cdots +$$
$$+ P(3{,}333 \le X \le 5) \cdot \left(\frac{(3{,}333+4)}{2} - 1 \right) + P(X \ge 5) \cdot (4-1)$$

This premium will not be exact since it is just an interpolation between the values in the empirical distribution. The equation value can only be used as an approximate premium for the layer 3 excess of 1.

4.2 Adjustment

The adjustment must be as good as possible for the accurate interval. After all contracts were transformed to the 28 unit, the excess point was added to the limit for all contracts. I noticed that the adjustment must be good in the interval from a loss amount at 1 to 3.33, between the years 3 to 20.

The adjustment of different distributions gave the lowest difference with the Pareto distribution with parameters (4.00, 4.92), truncated at 28. That distribution has 17.00 years between Mireille, which is the same as Skandia Internationals assumptions, and when I looked at the pure premium the approximate premium from the *EF* is 99.75% of the calculated pure premium.

The Pareto distribution used in the project has the following notation, parameters and distribution function.

$$Pareto: \quad X \in Pa$$

$$F(x) = 1 - \left(\frac{\alpha}{\alpha + x} \right)^{\gamma} \qquad \alpha > 0 \; \gamma > 0,$$

$$if \; \gamma > 1 \Rightarrow \exists \, E[X], \qquad if \; \gamma > 2 \Rightarrow \exists \, Var(X)$$

If m is an upper truncation level

$$G(x) = \begin{cases} F(x)/F(m) & x < m \\ 1 & x \ge m \end{cases}$$

5. Calculation of pure premiums

In this chapter the equations and formulas which are needed to calculate the pure premium for one layer will be solved, starting with the pure premium and ending with the reinstatement premium.

5.1 Expected loss for a layer

If we suppose that the loss amount is the random variable, X, and denote the excess-point with s and limit with l. If a windstorm occurs with a loss amount of X, then the reinsurance treaty will be affected with the following amount: Max {0, min {X-s, l}}.

Definition: Pure premium

The pure premium, r, for a layer is the expected loss to that layer.

The formula for the pure premium is; $r = E$ [max {0, min {X-s, l}} | X<m].

Theorem: Pure premium for an Excess of Loss contract with Pareto distributed loss amount.

Suppose that the Excess of Loss layer is l excess of s and that there exist a maximal loss amount, m. Suppose that the loss is Pareto (α,γ) distributed and $\gamma>1$. Then the pure premium, r, is:

$$r = \frac{1}{F(m)} \cdot \left(l \cdot (F(m) - 1) + \frac{\alpha^{\gamma}}{(-\gamma+1)} \left((\alpha + s + l)^{-\gamma+1} - (\alpha + s)^{-\gamma+1} \right) \right).$$

Proof

The proof of this theorem is quite straitforward but in this article it is not necessary to go through every detail. I will only describe the first step and for a full proof I refer to my original Report (Stockholm University, ISSN 0282-9169).

I start to solve the minimum and maximum functions and after that I used a Lemma with the surviving function.

Lemma: Expected value for a nonnegative random variable Y,

$$E[Y] = \int_0^{\infty} (1 - F(y))\, dy$$

In the lemma I needed the surviving function of the loss in one layer.

$$P(X - s \le x | s \le X \le s + l) = \frac{P(X - s \le x, X - s \le l | s \le X \le m)}{P(X - s \le l | s \le X \le m)} =$$
$$= \frac{F(x+s) - F(s)}{F(s+l) - F(s)}, \quad 0 \le x \le l$$

When this is done the rest is ordinary mathematical calculations.

5.2 Reinstatement premium

All the analysed contracts have the same reinstatement terms - one full reinstatement at 100% additional premium as to time, pro rata as to amount. The total premium for the layer will be the additional premium, P, added with the reinstatement premium. The expected reinstatement premium will be the additional premium times the pure premium as a share of limit. If this share is s, then the total premium will be the additional premium multiplied with 1+s.

$$P_{tot} = P + P \cdot s = P(1+s)$$
$$Q(t) = \frac{Rate\ with\ reinstatemen\,(t)}{Pure\ rate\,(t)} = \frac{Rate(t)}{Pure\ rate(t)/(1+s)}$$

5.3 Variance calculation of the risk premium

Another measure for the layer (l excess of s) is the variance and I have calculated that measure, but since I do not use that in the mathematical model I do not treat that part of my analysis in this article except of an example. Again I assume the event loss to be Pareto (4, 4.9) and we consider a layer l excess of s.

$$r = E[\max\{0, \min\{X\text{-}s, l\}\} | X \le m] =$$
$$= 0 \cdot P(X - s < 0 | X \le m)) + E[\min\{X - s, l\} | s \le X \le m] \cdot P(X - s \ge 0 | X \le m) =$$
$$= (1 - G(s)) \cdot \langle E[X - s | s \le X \le s + l] \cdot P(X \le s + l | s \le X \le m) + l \cdot P(X \ge s + l | s \le X \le m) \rangle$$

The table below shows that a layer between 1 and 3, in the same unit as the distribution adjustment, i.e. maximum loss amount is 28. There are some initial conclusions that can be made from the table, e.g.:

- the sum of the expected values of the two layers 1 excess of 1 and 1 excess of 2 is the same as the layer 2 excess of 1 (as it should be)
- it makes a palpable difference to shift the layer upwards both in the expected value and the variance.

6. Mathematical model of rate variations

The purpose of the project was to set up a mathematical model for how the reinsurance premium for a Windstorm Excess of Loss reinsurance fluctuates over time; i.e. establish relation between the market premium and the last major windstorm.

6.1 Background

A mathematical model of how the *MRI* changes over time has to be as simple as possible, otherwise it will be impossible to estimate the parameters.

It also has to describe the reality in the best way. Because of the global reinsurance market, this will be very difficult if only a few parameters are used.

Probably, the rates (prices) of reinsurance depends on:

- The risks and values.
  The value of the risks and the probability of loss
- Competition on local market.
  Prices for direct insurance, etc.
- Competition on the global market.
  Free capacity on the reinsurance market, number of interested reinsurers, etc.
- Last major storm in Japan.
  Time since the last one, size of the loss, how many layers were affected, risk assessment changes, etc.
- Last major storm or catastrophe in the world.
  Time since the last major event, size of the loss, how many companies were affected and changes in their capacity for windstorm reinsurance (are they more moderate), etc.
- State of the market.
  State of reinsurance market in Japan, Asia and World, state of other markets related to the insurance (reinsurance), etc.

There are more factors that make the rate fluctuate and those mentioned above are just a small selection in order to describe how complex the model really would have to be.

6.2 Mathematical model

In this part I will discuss how a multiplicative model for the premium can look like.

I start to assume that the model for the Rate (Premium for layer/Premium Base) year t and when the last major windstorm were in year k, is

$P(t, k)$ = Pure rate $(t) \times MRI\ (t, k) \times \varepsilon(t)$.

This make the ratios, $Q_i(t)$, to be observations from the $MRI\ (t, k) \times \varepsilon(t)$ and $\varepsilon(t)$ is a random dispersion variable.

6.2.1 Japanese Market Rate Index model

My assumed mathematical model for the Market Rate Index is:

Limit	*Xs-point*	*25 % quartile*	*Expected value = r*	*75 % quartile*	*Variance*	*St. dev. SD*	*Years X>s+l*	*COV SD / r*
2	*1*	0.235	0.311	1.044	1.812	1.346	15.72	4.325
1	*1*	0.165	0.217	0.634	1.591	1.261	7.36	5.812
1	*2*	0.176	0.094	0.653	0.927	0.963	15.72	10.212
2	*1.2*	0.241	0.263	1.060	1.616	1.271	18.06	4.841

$$MRI(t,k) = Z(t) \cdot C(t,k) =$$
$$= \left(v_0 + v \cdot \cos\left(\frac{2\pi t}{T} + 2\pi\left(1 - \frac{T_{\max}}{T}\right)\right)\right) \cdot \left(1 + \exp\{-(t-k) \cdot S\}\right)$$

Where I assume that the *MRI* is depending on state on market, *Z*, and with a loading, *C*, that depends on in which year the last major windstorm occurred.

I also assume that, $Z(t)$, the state of the market parameter, has a cyclic appearance (maybe with a random period and amplitude); $Z(t) = v_0 + v \times \cos(\Omega t + \Theta)$. The last state of the market maximum were T_{max} (fixed) and the period length is T (maybe stochastic).

The load, $C(t, k)$, is depending on last major windstorm and it is a decreasing continuous function. The function, $C(t, k)$ are depending on a function, S, that is depending on the magnitude (loss amount) of the last windstorm (inverse related). When the loss amount is increasing S will decrease; $C(t,k) = 1 + \exp\{-(t-k) \cdot S\}$.

7. Analysis results

The average ratio results ($Q_i(t)$, including the reinstatement premium) for every year is shown in the table below. Please note that this average is the estimate of the MRI for each year. I have included the standard deviation to show how spread the different outcomes are.

Parameter estimations

	Average	*Std.dev.*
1992	1,16	0,25
1993	1,19	0,25
1994	1,09	0,24
1995	0,95	0,25
1996	0,78	0,20
1997	0,59	0,14

We have an estimation of the Japanese Market Rate Index, MRI, for the years 1992 to 1996. To get the Rate for a contract we have to estimate the random parameter, ε.

Is this parameter depending on the state of the market? When the competition is hard (like now) the differences between the prices seem to be decreasing. See the Average and Standard deviation for each year.

8 Conclusions

8.1 Result

We have now a better understanding of the complexity of the reinsurance market, and the difficulties Skandia International and their underwriters have when they shall underwrite a catastrophe Excess of Loss contract.

We know how to calculate the risk premium and variance for an Excess of Loss layer when the losses are Pareto distributed. As we have noted, a small change in excess point will make a palpable change of the risk premium and variance.

We have a mathematical model for the *MRI* fluctuation but we can not estimate and verify the parameters of the model.

The conclusion is that it is very difficult to statistically build a mathematical model for the premium fluctuations. A better model could be built and statistically verified if the information was more comprehensive.

In the analysis I have used the same loss distribution for all years 1991 until 1997. It shall be noted that the price depends on the information from previous years and not on coming years, so the loss distribution will not be the same for all previous years. This implies that the adjustment of the loss distribution should be recalculated every year based on the new information.